Description of a Reference Material for the

Calibration of Optical Systems for Dynamic and Static Deformation Measurement

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Calibration of Optical Systems
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This protocol is based on the "Guidelines for the Calibration of Optical Systems for Dynamic Deformation Measurement" that were prepared as part of the ADVISE project (Grant Agreement SCP7-GA-2008-218595) within the European Commission's Safety and Security by Design Programme (FP7-SST-2007-RTD-1). It has been adapted in the VANESSA Coordination and Support Action (Grant Agreement FP7-NMP-2012-CSA-6).

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1. Description of Reference Material

1.1 Design

The design of the Reference Material consists of a simple cantilever in the form of a stepped bar as shown in Figure 1. The cantilever has thickness T, length 40T, and width 10T, with at one end an enlarged portion of thickness 5T and length 20T, giving an overall length of 60T.

The origin of coordinates is at the intersection of the axis of symmetry of the Reference Material with the section change, such that *x* is measured along the cantilever towards the tip, *y* across the face of the cantilever, and *z* from the neutral axis through the thickness.

The enlarged portion is used for clamping the cantilever to a rigid, immovable body. Experiments have shown that the behaviour of the cantilever is independent of the clamping method and force, provided there is no relative movement between the enlarged end of the Reference Material and the rigid, immovable body.

The design is scalable and the dimensions and materials should be chosen so that deformations are comparable to those it is expected to measure with the calibrated measurement system. However, the Reference Material should remain in the elastic loading regime so that the process is reproducible.

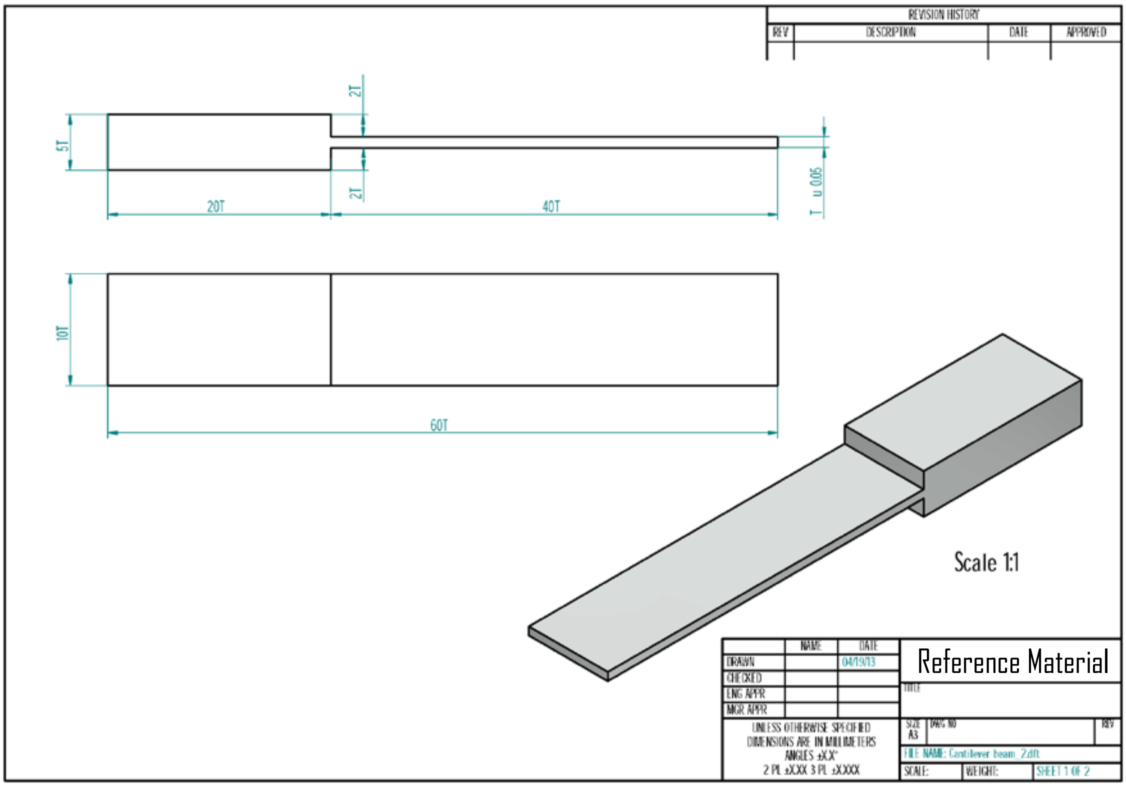

Figure 1: Drawing and three-dimensional rendering of the Reference Material.
The thick section is intended to be rigidly clamped to a large, immovable body.
Manufacturing guidelines are given in Appendix A.

1.2 Natural frequencies

The natural frequencies, f_k for the resonant bending modes of the Reference Material can be found analytically as[1]

$$f_k = \frac{\lambda_k^2}{2\pi L^2}\left(\frac{EIL}{M}\right)^{1/2} \tag{1}$$

where L is the length of the cantilever, E is the modulus of elasticity, I is the second moment of area and M is the mass of the cantilever, which in this case is $2M_{RM}/7$ where M_{RM} is the mass of the complete Reference Material. The second moment of area is defined as $I=bh^3/12$, which for the Reference Material gives $I=5T^4/6$.

The dimensionless natural frequency parameters, λ_k, are computed from

$$\cos\lambda \cosh\lambda + 1 = 0 \tag{2}$$

The first few values are given in **Table 1**. For k > 4 use $\lambda_k = (2k-1)\frac{\pi}{2}$ and ϕ_k =1.

Table 1: Mode shape parameters of a cantilever beam clamped at one end.

λ_1 = 1.875	λ_2 = 4.694	λ_3 = 7.855	λ_4 = 10.996
λ_1^2 = 3.516	λ_2^2 = 22.03	λ_3^2 = 61.70	λ_4^2 = 120.9
$\phi_1 = 0.7340$	$\phi_2 = 1.0185$	$\phi_3 = 0.9992$	$\phi_4 = 1.0000$

1.3 Deformation field equation for resonant bending modes

The modal shape for bending mode k is given by (Appendix B)

$$w_k\left(\frac{x}{L}\right) = \frac{\delta}{2}\left\{\cosh\lambda_k\frac{L-x}{L} + \cos\lambda_k\frac{L-x}{L} - \phi_k\left(\sinh\lambda_k\frac{L-x}{L} + \sin\lambda_k\frac{L-x}{L}\right)\right\} \tag{3}$$

where w_k is the out-of-plane displacement of the cantilever in bending mode k, δ is the tip amplitude, x is measured from the clamped end along the cantilever, and ϕ_k is given by

$$\phi_k = \frac{\sinh\lambda_k - \sin\lambda_k}{\cosh\lambda_k + \cos\lambda_k} \tag{4}$$

The direct strain along the cantilever, ε_x is given by

$$\varepsilon_x(x,y) = z\frac{d^2w(x)}{dx^2} \tag{5}$$

where z is the distance from the neutral axis through the thickness of the cantilever beam, such that at the top surface $z = T/2$. The surface strain is given by

$$\varepsilon_k(x,y) = \delta\frac{T\lambda_k^2}{4L^2}\left\{\cosh\lambda_k\frac{L-x}{L} - \cos\lambda_k\frac{L-x}{L} - \phi_k\left(\sinh\lambda_k\frac{L-x}{L} - \sin\lambda_k\frac{L-x}{L}\right)\right\} \tag{6}$$

[1] Blevins, R.D., 2001, *Formulas for natural frequency and mode shape*, Krieger Pub. Co., Malabar, FA.

The components of surface strain in the *y*-direction are zero except within a distance equivalent to the width of the cantilever (W=10T) from the clamp.

Figure 2 shows the analytic resonant bending mode shapes for a cantilever with a clamped and a free end. Note that the maximum deflection occurs at the tip for all modes.

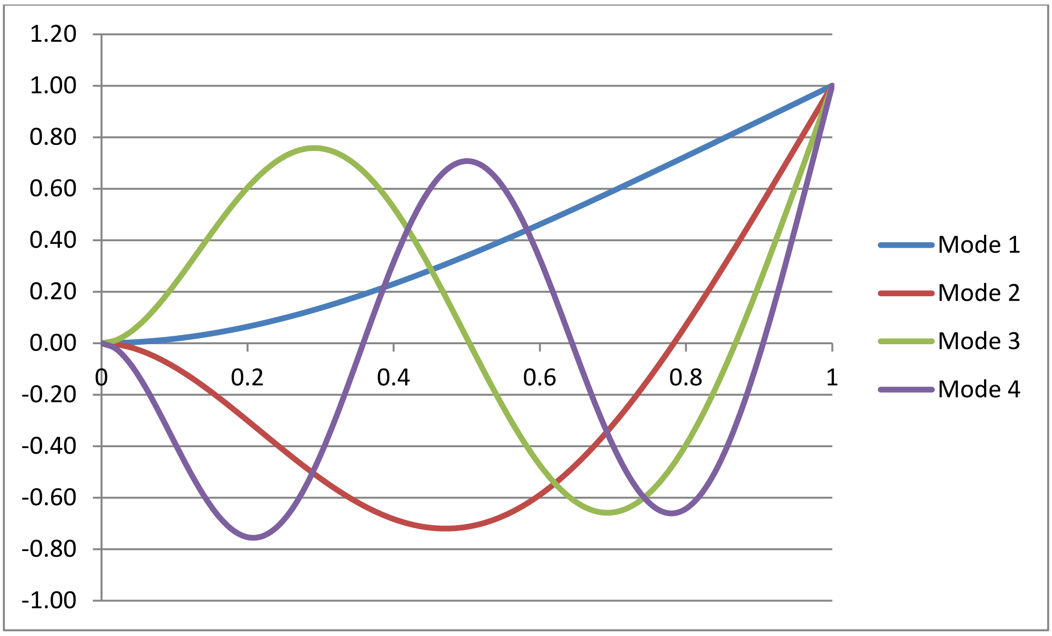

Figure 2: First four normalized bending modes of a cantilever clamped at one end.

1.4 Deformation field equation for static load

When the cantilever is deformed by a load *P* acting at its tip, then, in the linear regime, the out-of-plane displacement is given by [2]

$$w(x) = \frac{Px^2}{6EI}(3L - x) \tag{7}$$

If the displacement at the tip, $v(L)=\delta$ is known then

$$P = \frac{6EI \times \delta}{2L^3} \tag{8}$$

and expression (7) can be rewritten as

$$w(x) = \frac{\delta}{2L^3}\left(3Lx^2 - x^3\right) \quad \text{or} \quad w\left(\frac{x}{L}\right) = \frac{\delta}{2}\left(3\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)^3\right) \tag{9}$$

Using Eq. (5), the direct strain along the cantilever, ε_x, is given by

$$\varepsilon_x(x, y) = \delta\frac{3T}{2L^2}\left(1 - \frac{x}{L}\right) \tag{10}$$

The component of strain on the surface in the *y*-direction is zero except within a distance equivalent to the width of the cantilever (*W*=10T) from the clamp and location of the applied static load.

[2] Young, W.C., 1989, Roark's formulas for stress & strain, 6th ed., McGraw-Hill Book Co., New York

Appendix A: Manufacturing guidelines

The design consists of a simple cantilever in the form of a stepped bar as shown in Figure 1 and Figure 3. It is manufactured from a single piece of any homogeneous, isotropic material that is free of residual stress. The design of the Reference Material is parametric with a cantilever of thickness T, width 10T, and length 40T, with at one end an enlarged portion of thickness 5T and length 20T, giving an overall length of 60T. The manufacturing tolerances should be 0.05mm. The fillet radius at the transition between the cantilever and the enlarged portion should be as small as it is practical to manufacture, in order to minimise the effect of the strain distribution in the cantilever.

The design is scalable, and the dimensions and materials should be chosen so that deformations are comparable to those it is expected to measure with the calibrated measurement system; in addition, the Reference Material should occupy the majority of the field of view of the optical arrangement set-up for the planned experiments.

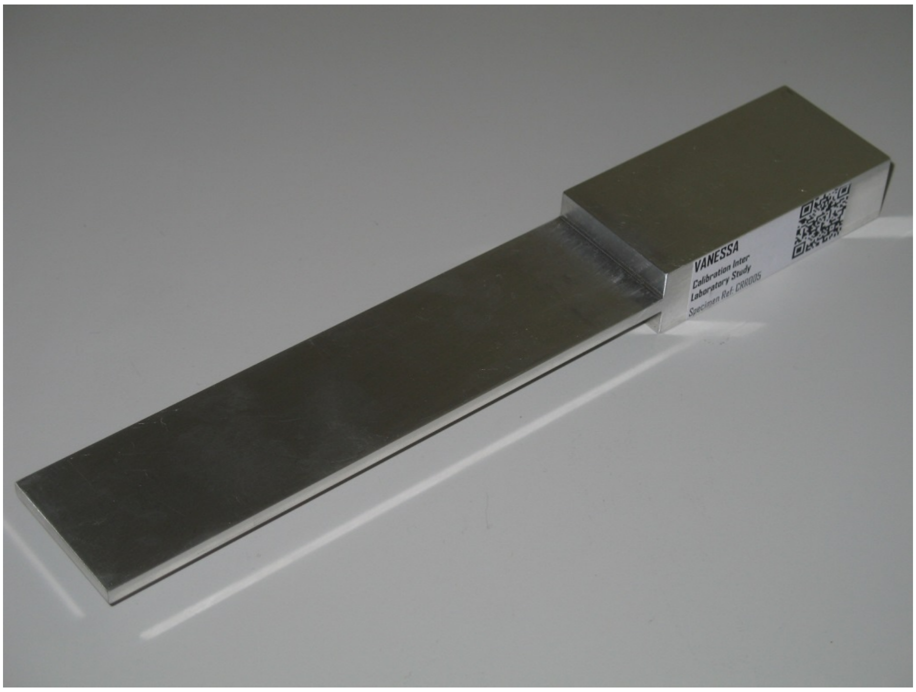

Figure 3: Reference Material used in the Interlaboratory Study.